The International Trade as the Sole Engine of Growth for an Economy

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Running title: Trade as the Sole Engine of Growth
Can international trade act as the sole engine of growth for an economy? If yes, how?

This paper answers these questions with two simple two-country models of exogenous growth, in which just one country enjoys sustained growth in autarky. The models differ in the assumptions on technical change, which is either labor- or capital-augmenting. In both cases, the stagnant economy imports growth by trading. In the first model, growth is transmitted because of permanent increases in the trade volume. In the alternative framework, the stagnant economy imports sustained growth because its terms of trade permanently improve. The simple models can be generalized, while preserving these results.

Key words: international trade; stagnant economies; growth transmission; mechanisms of transmission

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1. INTRODUCTION

The positive impact of international trade on economic growth has been widely documented from both a theoretical (e.g. Grossman and Helpman, 1997) and empirical (e.g. Keller, 2002) point of view. To this respect, it has been argued that openness is important for growth because it generates channels for technology diffusion, which makes possible for less developed countries to import productivity gains from overseas.

The literature on trade and growth has paid special attention to technological spillovers as channels of diffusion (Keller, 2004). Developing countries learn from imported technology and also from technological progress embodied in imported goods. This learning increases domestic stock of knowledge and, hence, domestic productivity and growth. Thus, one could argue that the greater the trade volume (imports and exports) is, the more knowledge can be potentially accumulated. However, the access to lower prices may also constitute a way of importing foreign productivity gains. Technological progress makes possible to produce goods of increasing quality at each time lower costs. Thus, there is also a trade benefit from having access to more productive intermediate and capital goods at a lower cost than their opportunity cost, which includes the one associated to the necessary R&D to develop them.

Therefore, international trade can potentially play a crucial role in fueling economic growth of less developed countries. In other words, it can become one of the engines of growth for a country. But, going a step further, might a stagnant economy import sustained growth by trading? An affirmative answer would be an argument in favor of views on trade as a way to escape from poverty. If this is the case, what are the mechanisms through which trade could transmit sustained growth? The above discussion seems to suggest that trade could operate via trade volume and/or via prices.
Our goal in this paper is to offer theoretical answers to these questions. To do that, we develop two simple two-country models of exogenous growth and trade in intermediate goods that build on Ventura’s (1997). The models are identical in everything except in the type of technical progress, which is either labor- or capital augmenting (model 1 and 2, respectively). Since we are concerned with the transmission of sustained growth, our analysis concentrates on the long-run equilibrium.

Countries produce one non-traded final good with constant returns to scale technologies that use two intermediate goods as inputs. Intermediates are produced with linear technologies that use either labor (good 1) or capital (good 2) as factor inputs, and can be potentially traded in international markets. The economies only differ regarding the existence of technological progress. More specifically, the total factor productivity (TFP) is positive in one of the countries (country 1), while it is equal to zero in the other economy (country 2). In the first framework, the presence of labor-augmenting technical change in good 1 sector in country 1 amounts to the condition that labor is a reproducible factor. Thus, technologies of intermediate goods 1 and 2 in country 1 and good 2 in country 2 are characterized by constant returns to scale in reproducible factors, while the technology of good 1 in country 2 exhibits constant returns to scale in labor, a non-reproducible factor. In model 2, technical change is capital-augmenting and, hence, labor becomes a non-reproducible factor in the two economies.

In the autarky situation, the first economy enjoys permanent growth, while the growth rate of the second one is nil. At long-run, countries trade following their comparative advantage. No matter the assumption on technical change, growth is transmitted from country 1 to country 2. However, the mechanism of transmission sharply differs in both models. In model 1, the terms of trade emerge as the variable linking growth rates in both countries, but they converge to a constant value at long-run.
The stagnant economy overcomes decreasing returns to capital accumulation because it imports (exports) increasing amounts of good 1 (good 2) from country 1 (to country 1). Therefore, growth is transmitted because of permanent rises in the trade volume. In model 2, country 2 does not accumulate capital at long-run. The evolution of the terms of trade, instead of increases in the trade volume, constitutes the mechanism of transmission. Though the exported amount of intermediate input by country 2 remains constant, its imports raise over time because the terms of trade become increasingly favorable to this economy.

It is worthwhile to notice that the behavior of international relative prices depends on the world relative scarcity of intermediate inputs. In model 1, the terms of trade of countries tend to a constant value because the world production of goods 1 and 2 tends to sustainably grow at the same rate. This result comes from the fact that international trade allows country 2 importing good 1, which is produced with a technology that exhibits constant returns to scale in reproducible factors. This is the reason why country 2 overcomes decreasing returns and can permanently accumulate capital. Hence, trade operates via trade volume. In model 2, country 2’s terms of trade permanently improve because good 1 becomes increasingly scarcer than good 2. In this case, country 2 does not need to accumulate capital for continuously raising its consumption level. Thus, trade operates via relative international prices.

Of course, our simple theoretical frameworks can be generalized in several ways to include more realistic assumptions, as more general technologies, endogenous growth and other sectors, while preserving the main results. In this respect, we propose two models as examples.

The theoretical literature on this issue is rather scarce. Findlay (1980) developed a two-country model of exogenous growth to show that trade can act as the only engine of
growth for an economy. As in our model 1, the terms of trade are constant at long-run, while the trade volume permanently increases. However, his model is more restrictive than ours is, inasmuch as it assumes constant saving rates and fixed wage in one of the countries.\footnote{In his model, the developed North is represented by Solow’s (1956) framework, and the less developed South behaves as Lewis’ (1954) economy.} The study by Acemoglu and Ventura (2002) is also related to our model 1. They found that, even in absence of diminishing returns in production and technological spillovers, trade leads to a stable world income distribution. This is because countries that accumulate capital faster than the world average undergo declining terms of trade, which restrains incentives for further accumulation. At long-run, the terms of trade become constant and countries grow at the same rate. Though they were not directly interested with the transmission of sustained growth through trade, their model also has the implication that trade can emerge as an engine of growth for stagnant economies.

Diewert and Morrison (1986) aimed at developing an empirical method for properly measuring the contribution of factor inputs to output growth of open economies. They proved, with an empirical model based on index numbers, that an increase in the price of exports relative to imports has an effect that is similar to an increase in TFP.

As indicated at the beginning of this section, the causality going from trade to growth is a well-established empirical result. Ekholm and Södersten (2002) emphasized the importance of considering income-terms of trade\footnote{In his model, the developed North is represented by Solow’s (1956) framework, and the less developed South behaves as Lewis’ (1954) economy.} when analyzing the relationship between trade and growth. Looking at data, they found that the terms of trade presented a roughly constant trend, while income-terms of trade increased over time. Thus, they argued that openness relates to growth mainly through the trade volume. Several studies, as those by Frankel and Romer (2002) and Alcalá and Ciccone (2004), have obtained a significant causality going from the trade volume to TFP, and hence to growth. Regarding the second mechanism of transmission, the growth regressions by
Barro (1991) showed that the economic growth is significantly positively related to the terms of trade. Kohli (1997) and (2004) exploited Diewert and Morrison (1986) result to show that the omission of terms of trade movements may seriously under- and over-estimate TFP and economic growth when the terms of trade improve and deteriorate, respectively.

We cannot close this short survey of the related literature without commenting the observed deterioration of less developed countries’ terms of trade (e.g. Sarkar and Singer, 1991), since it can be interpreted as against the hypothesis of growth transmission through the terms of trade. Nevertheless, this finding has received two important criticisms. Firstly, because of the lack of statistical information, most researchers do not adjust prices for changes in the quality of goods. Since an increase in the quality of imports relative to exports leads to an improvement in the terms of trade, this omission might seriously understate less developed economies’ terms of trade (Athukorala, 1993). Secondly, the observed deterioration refers to developing countries as a group. However, Athukorala (2000) argued that these economies are far from being homogeneous and, hence, what holds for the group might not occur for some countries. This author showed that this is the case of the Sri Lankan economy.

The remainder of this paper is organized as follows. Section 2 describes the models, while Section 3 solves for the autarky equilibrium. Section 4 characterizes the trade situation and shows that sustained growth is transmitted from country 1 to country 2, though the mechanism of transmission differs in both models. Section 5 proposes two examples of more general models that deliver the same type of results. Lastly, Section 6 summarizes and concludes.

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2 This measure is defined as the product of export volume and net barter terms of trade.
2. THE MODELS

The models that we develop build on Ventura’s (1997), and differ regarding the type of technical change. Time is continuous and endless and the world economy consists of two large countries, \( i = 1,2 \). There is a non-traded final good, \( y^i(t), i = 1,2 \), that can be used for consumption or investment; two potentially traded intermediate goods, \( x^i_{zt}(t), z = 1,2 \), (good 1 and 2) produced in country \( i \) and used as inputs in the final good production of country \( j \); and two factors of production, capital \( k^i(t) \) and labor \( l^i(t) \), allocated to intermediate goods productions. The notation regarding the production of intermediate inputs needs further clarification. The total production of intermediate input \( z \) in country \( i \) will be denoted by \( x^i_{zt}(t) \). The second sub-index will only appear if a part of the total production is exported. In addition, we assume that international factor flows are not allowed, all markets are competitive and foreign and domestic intermediate goods are perfect substitute for each other.

Each country is inhabited by a continuum of identical households that is normalized to the unit. There is no population growth. Households are endowed with one unit of time at every period that can only be allocated to work. These assumptions imply that the population amounts to the labor force of the economies, and that all variables are expressed in per capita terms.

Countries possess a common final good technology that is represented by a Cobb-Douglas production function with constant returns to scale:

\[
y^i(t) = \left( x^i_{zt}(t) \right)^{\alpha} \left( l^i(t) \right)^{1-\alpha}, \quad 0 < \alpha < 1. \tag{1}
\]

Similarly to Ventura’s (1997) model, good 1 and good 2 are produced with labor and capital, respectively, through the following linear production functions:
\[
x_i(t) = A_i(t) f(t) ; \quad \dot{x}_i(t) = B_i(t) k(t),
\]
\[
A^2(t) = B^2(t) = I \forall t,
\]
Model 1: \( A^1(t) = e^{\gamma t}, \gamma > 0 \) and \( B^1(t) = I \forall t,
\]
Model 2: \( A^1(t) = I \forall t \) and \( B^1(t) = e^{\gamma t}, \gamma > 0, \)

where \( A^i(t) \) and \( B^i(t) \) represent technological progress in good 1 and 2 sectors, respectively. From now on, we will assume that \( A^2(t) = B^2(t) = I \forall t, \) which implies that country 2 does not enjoy sustained growth in autarky. The models only differs regarding the assumptions on \( A^i(t) \) and \( B^i(t) \). In the first model, technological progress in country 1 is labor-augmenting and thus \( A^1(t) = e^{\gamma t}, \gamma > 0 \) and \( B^1(t) = I \forall t. \) In the alternative framework, the assumptions are \( A^1(t) = I \forall t \) and \( B^1(t) = e^{\gamma t}, \gamma > 0 \) and, hence, technical change is capital-augmenting.

A general result under a Cobb-Douglas production function is that labor- and capital-augmenting technical changes are in essence the same. Both of them allow for the existence of a balance growth path (BGP). However, these two approaches lead to different results in our models, since they affect the mechanism through which trade operates in transmitting sustained growth. Notice that the existence of labor-augmenting technical progress amounts to the condition that labor is a reproducible factor in country 1. Thus, in this country both intermediate goods are produced with technologies that exhibit constant returns to scale in reproducible factors. In country 2, however, labor is a non-reproducible factor, while the production function of good 2 exhibits constant returns to scale in capital. In the model with capital-augmenting technical change, the labor input is a non-reproducible factor in both economies.
Countries do not differ regarding preferences. The representative household derives utility from the consumption of the aggregate good, $c^i(t)$, and maximizes its intertemporal utility discounted at a positive rate $\rho$:

$$U^i(0) = \int_0^\infty e^{-\rho t} \left( \frac{c^i(t)}{t^\theta} - 1 \right) dt, \quad \theta > 0,$$

subject to the budget constraint and the initial endowment of wealth:

$$\dot{a}^i(t) = r^i(t) a^i(t) + \bar{w}^i(t) - c^i(t),$$

$$a^i(0) > 0 \text{ given}$$

where $a^i(t)$ denotes wealth, $r^i(t)$ is the interest rate and $\bar{w}^i(t)$ is the wage per time unit. As it is usual, we assume that agents have perfect foresight.

### 3. THE AUTARKY SITUATION

In autarky countries behave as the well-known Ramsey model, but with exogenous growth in the case of country 1. The profits-maximizing behavior of firms in the final good sector implies that intermediate good prices equal marginal productivities:

$$p_1^i(t) = \alpha \left( x_1^i(t) \right)^{\alpha - 1} \left( x_2^i(t) \right)^{1 - \alpha} \left( x_3^i(t) \right)^{\gamma \alpha} \quad \rightarrow \quad \frac{p_1^i(t)}{p_2^i(t)} = \frac{\alpha \ x_2^i(t)}{1 - \alpha \ x_3^i(t)},$$

where $p_z^i(t), \ z = 1, 2$ denotes the price of good $z$ in country $i$. From now on, we will refer to the price of good 1 relative to good 2 as relative price. The price of final good is taken as numerary. In intermediate goods sectors, wage and the interest rate equal labor and capital marginal productivities, respectively:

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3 If $\theta = 1$, current utility becomes $\ln c^i(t)$.
where we assume that capital depreciates at the same rate $\delta > 0$ in both economies.

The competitive equilibrium is as a set of allocations and prices that satisfy firms and household problems, and that clear all markets in both economies, including capital market, $a_t^i(t) = k_t^i(t)$, $i = 1, 2$, and labor market, $l_t^i(t) = 1$, $i = 1, 2$.

Next, we characterize the autarky equilibrium under the two hypotheses on technological change.

3.1. Model 1: Labor-Augmenting Technical Change

Given the assumptions in the previous section, country 1 will grow at the rate $\gamma$ at long-run, while country 2 will not enjoy sustained growth. More concretely, looking at the interest rate of countries:

$$r_t^1(t) = (1 - \alpha) \left( \frac{k_t^1(t)}{e^{\gamma t}} \right)^{-\alpha} - \delta, \quad r_t^2(t) = (1 - \alpha) \left( k_t^2(t) \right)^{-\alpha} - \delta,$$

it follows that the long-run equilibrium of country 1 is characterized by a BGP, while that of country 2 is a steady state (the long-run growth rate is equal to zero).

The equilibrium condition for choosing consumption over time (Euler equation governing consumption) evaluated in the long-run equilibrium permits to obtain relative prices:

$$\frac{p_t^1(t)}{p_t^2(t)} = \frac{\alpha}{1 - \alpha} \frac{k_t^1(t)}{e^{\gamma t}} \xrightarrow{\text{BGP}} \frac{p_t^1}{p_t^2} = \alpha (1 - \alpha)^{\frac{\rho}{\gamma + \rho + \delta}} \left( \frac{1}{\gamma + \rho + \delta} \right) \frac{1}{\gamma},$$

$$\frac{p_t^2(t)}{p_t^2(t)} = \frac{\alpha}{1 - \alpha} \frac{k_t^2(t)}{e^{\delta t}} \xrightarrow{\text{BGP}} \frac{p_t^2}{p_t^2} = \alpha (1 - \alpha)^{\frac{\rho}{\rho + \delta}} \left( \frac{1}{\rho + \delta} \right) \frac{1}{\gamma}. $$

$$w_t^i(t) = p_t^i(t) A_t^i(t); \quad \dot{r}_t^i(t) = p_t^i(t) B_t^i(t) - \delta, $$

(6)
Here and throughout the paper, the omission of time will denote stationary values over the long-run equilibrium. The relative price of country 1 and 2 becomes constant as time passes. In country 1, this result is due to the assumption of constant returns to scale in reproducible factors. In country 2 the absence of technical progress is the reason why the relative price becomes stationary.

The autarky prices in (8) will be useful later on for establishing the comparative advantage of countries, and the conditions that justify the existence of international trade.

3.2. Model 2: Capital-Augmenting Technical Change

The results regarding country 1 depend on the assumption on technical change. In the case of capital-augmenting technical progress, the interest rate in country 1 equals:

\[
 r^1(t) = (1 - \alpha) \left( \frac{k^1(t)}{e^{\alpha \gamma t}} \right)^{-\alpha} - \delta, \tag{9}
\]

Interest rate must be constant over the BGP, which requires that capital and, hence, final good output and consumption grow at the same constant rate \( \dot{\vartheta} \):

\[
 \dot{\vartheta} = \frac{1 - \alpha}{\alpha} \gamma. \tag{10}
\]

The relative price is obtained by proceeding as in the previous subsection:

\[
 \frac{p^1(t)}{p^2(t)} = \frac{\alpha}{1 - \alpha} e^{\nu t} k^1(t) \xrightarrow{\text{BGP}} \frac{p^1(t)}{p^2(t)} = \alpha \left( 1 - \alpha \right)^{\frac{1 - \alpha}{\alpha}} \frac{\alpha e^{\gamma t}}{(1 - \alpha \gamma \theta + \alpha (\rho + \delta))}. \tag{11}
\]

Unlike the case with labor-augmenting technological progress, country 1 relative price at long-run strictly increases through time at the rate \( \gamma / \alpha \). The reason is quite
obvious: good 1 is produced with a non-reproducible factor, and the production of good 2 grows at the rate $\gamma/\alpha$.

4. THE FREE TRADE SITUATION

Given that intermediates are produced with just one of available factors in each economy, the existence of trade will imply that it is characterized by incomplete specialization of both countries. Notice that countries will always use their labor and, hence, will always produce good 1. However, not to accumulate physical capital might become optimal for one of the countries, since imports from the other economy can act as a substitute for capital accumulation.

Our objective in the next subsections is to answer the question on whether international trade is able to transmit permanent growth to country 2. We start by establishing the specialization pattern of countries. Then, we show that trade acts as a transmitter of permanent growth whatever is the assumption on technical change. Nevertheless, we find that this assumption matters for how growth is transmitted.

4.1 Model 1: Labor-Augmenting Technical Change

From relative prices in expression (8), it is clear that country 1 and 2 eventually have comparative advantage in the production of intermediate good 1 and 2, respectively. The structure of the model implies that investment in country 1 might hit a corner. In this case, country 1 would only produce good 1 at long-run. Since our main results hold whatever is the type of solution, we opt by assuming the existence of interior solution. Also notice that, by construction, the trade situation in our model leads interest rate equalization among countries at every time period.
Given the comparative advantage of countries, the productions of final good by countries are equal to:

\[ y^j(t) = \left( x^j_1(t) \right)^{\alpha} \left( x^j_2(t) \right)^{1-\alpha}, \quad y^s(t) = \left( x^s_1(t) \right)^{\alpha} \left( x^s_2(t) \right)^{1-\alpha}. \] (12)

It is convenient to remember that the second sub-index does not appear when the input is produced and entirely used within the country. From the profit-maximizing behavior of firms in the final good sector in country 1 and 2, and the equilibrium in the trade balance, \( p_1(t)x^j_{12}(t) = p_2(t)x^j_{21}(t) \), we obtain the following equilibrium conditions:

\[ \frac{p_1(t)}{p_2(t)} = \frac{\alpha \ x^j_1(t) + x^j_{21}(t)}{I - \alpha \ x^j_1(t) + x^j_{21}(t)} = \frac{\alpha \ x^j_{22}(t)}{I - \alpha \ x^j_{22}(t) + x^j_{12}(t)} = \frac{x^j_{21}(t)}{x^j_{12}(t)}. \] (13)

Straightforward manipulations of conditions in expression (13) permit to calculate exported-imported proportions of intermediate goods, which yields:

\[ v^j(t) \equiv \frac{x^j_{11}(t)}{x^j_1(t)}, \quad I - v^j(t) \equiv \frac{x^j_{12}(t)}{x^j_1(t)}, \]

\[ v^j(t) = \frac{(I - \alpha) x^j_1(t) x^j_2(t)}{x^j_1(t) \left( x^j_1(t) + x^j_2(t) \right)} + \frac{x^j_2(t) + \alpha x^j_1(t)}{x^j_1(t) + x^j_2(t)} ; \]

\[ v^s(t) \equiv \frac{x^s_{21}(t)}{x^s_2(t)}, \quad I - v^s(t) \equiv \frac{x^s_{22}(t)}{x^s_2(t)}, \]

\[ v^s(t) = \frac{\alpha \left( x^s_1(t) x^s_2(t) - x^s_1(t) x^s_2(t) \right)}{x^s_2(t) \left( x^s_1(t) + x^s_2(t) \right)}. \] (14)

Taking into account (13) and (14), we find that international relative prices depend on the ratio of world production of intermediates:

\[ \frac{p_1(t)}{p_2(t)} = \frac{\alpha \ x^i_1(t) + x^i_2(t)}{I - \alpha \ x^i_1(t) + x^i_2(t)} \rightarrow \frac{p_1(t)}{p_2(t)} = \frac{\alpha \ k^i(t) + k^s(t)}{I - \alpha \ e^\nu + 1}. \] (15)

A necessary condition for the transmission of sustained growth is that benefits from free trade do not extinguish as time passes. In other words, the trade situation must last
forever. Trade will be mutually beneficial for countries if relative prices in (8) and (15) fulfill the following conditions:

\[
\begin{align*}
&\frac{p_1^1(t)}{p_2^1(t)} \leq \frac{p_1(t)}{p_2(t)} \Rightarrow \frac{k^1(t)}{k^1(t) + k^2(t)} \leq \frac{e^{\gamma t}}{e^{\gamma t} + 1}, \\
&\frac{p_2^2(t)}{p_2^1(t)} > \frac{p_1(t)}{p_2(t)} \Rightarrow \frac{k^1(t)}{k^2(t)} < e^{\gamma t} + 1, \text{ for } t \geq t^*,
\end{align*}
\]

where \( t^* \) is the period at which countries entered into the trade situation.

Next, we concentrate on the long-run equilibrium of the world economy. As it becomes clear from the Euler equation, interest rate equalization implies that consumption of both countries grows at exactly the same rate at every period. From the expression for the interest rate in equilibrium:

\[
r(t) = (1 - \alpha) \left( \frac{k^1(t) + k^2(t)}{e^{\gamma t} + 1} \right)^{\alpha} - \delta,
\]

it follows that the world economy converges to a quasi-balanced growth path (QBGP). A QBGP in the model is a competitive equilibrium in which the growth rates of capital and consumption asymptotically converge to \( \gamma \). Imposing a growth rate of \( \gamma \) for per capita consumption, we obtain that the relative international price, \( \frac{p_1(t)}{p_2(t)} \), asymptotically approaches to the long-run autarky price of country 1 in expression (8). Moreover, conditions in (16) hold in the long-run and, hence, trade is always mutually beneficial for countries.

As the time variable tends to infinity, proportions in (14) asymptotically approach to:

\[
\lim_{t \to \infty} \nu^1(t) = \alpha + \phi (1 - \alpha); \quad \lim_{t \to \infty} \nu^2(t) = \alpha,
\]

\[
\text{where } \phi \equiv \frac{k^1(t)}{k^1(t) + k^2(t)}.
\]
Notice that the limit of $v^i(t)$ in (18) also corresponds to country 1’s share in gross world income, which is composed of all world labor share, plus the proportion of world capital share that corresponds to its capital. Country 2’s income comes increasingly from capital because its labor productivity is constant. Thus, its share in gross world income is just the proportion of world capital share that corresponds to its capital. The value of $\phi$ depends on capital stocks that countries had in the first period of trade.

We then conclude that country 2 imports sustained growth from country 1 by trading in intermediate goods. The mechanism of transmission is related to trade volume and not to terms of trade movements. Rewritten physical capital accumulation of country 2 as:

$$k^2(t) = \left(1 + \left(1 - v^i(t)\right)e^{\eta t}\right)\left(\frac{I - \alpha}{\alpha} \frac{p_1(t)}{p_2(t)}\right)^{1-\alpha} - c^2(t) - \delta k^2(t), \quad (19)$$

it follows that country 2 imports (exports) increasing amounts of intermediate good 1 (good 2). At long-run both $v^i(t)$ and the international relative price approach to constant values, while imports of good 1 (exports of good 2) raise over time due to productivity gains in sector 1 in country 1.

4.2. Model 2: Capital-Augmenting Technical Change

From the relative prices in (8) and (11), countries 1 and 2 eventually have comparative advantage in good 2 and 1, respectively. As in the previous case, we start by obtaining some results that will be useful to analyze the long-run equilibrium. The final good production in each country equals:

$$y^1(t) = \left(x^i_1(t) + x^i_2(t)\right)^{1-\alpha} \left(x^i_1(t)\right)^{1-\alpha}; \quad y^2(t) = \left(x^i_2(t)\right)^{1-\alpha} \left(x^i_1(t) + x^i_2(t)\right)^{1-\alpha}. \quad (20)$$
The competitive behavior of firms in the final good sector in country 1 and 2, and the equilibrium in the trade balance, \( p_j(t)x_{j,2}(t) = p_j(t)x_{j,1}(t) \), permit to obtain three expressions for the relative price that equalize in equilibrium:

\[
\frac{p_j(t)}{p_2(t)} = \frac{\alpha}{1-\alpha} \frac{x_{j,1}(t)}{x_{j,2}(t)} + \frac{\alpha}{1-\alpha} \frac{x_{j,1}(t) + x_{j,2}(t)}{x_{j,1}(t)} = \frac{x_{j,2}(t)}{x_{j,1}(t)}. \tag{21}
\]

Proportions of imported-exported proportions of intermediates can be calculated from (21):

\[
\begin{align*}
\nu^1(t) & = \frac{x_{j,1}(t)}{x_{j,2}(t)} = \frac{(1-\alpha)x_{j,1}(t) + x_{j,1}(t)(x_{j,1}(t) + \alpha x_{j,2}(t))}{x_{j,1}(t) + x_{j,2}(t)}; \\
\nu^2(t) & = \frac{x_{j,1}(t)}{x_{j,2}(t)} = \frac{(1-\alpha)(x_{j,1}(t)x_{j,2}(t) - x_{j,1}(t)x_{j,1}(t))}{x_{j,1}(t)(x_{j,1}(t) + x_{j,2}(t))}. \tag{22}
\end{align*}
\]

Plugging proportions in (22) into (21), the relative price results to be:

\[
\frac{p_j(t)}{p_2(t)} = \frac{\alpha}{1-\alpha} \frac{e^{\gamma t}k^1(t) + k^2(t)}{2}. \tag{23}
\]

In this case, the condition for the benefits from free trade to hold over time becomes:

\[
\begin{align*}
\frac{p_j^1(t)}{p_j^2(t)} & \geq \frac{p_2(t)}{p_1(t)} \quad \frac{p_j^2(t)}{p_j^1(t)} < \frac{p_1(t)}{p_2(t)} \quad \frac{k^2(t)}{k^1(t)} \leq e^{\gamma t} \quad \text{for } t \geq T, \tag{24}
\end{align*}
\]

Now, we are ready to analyze the long-run equilibrium of the world economy. Differently from the case with labor-augmenting technical progress, trade does not lead to interest rate equalization among countries:

\[
\begin{align*}
r^1(t) & = (1-\alpha) \left( \frac{e^{\gamma t}k^1(t) + k^2(t)}{2} \right)^\alpha e^{\gamma t} - \delta, \\
r^2(t) & = (1-\alpha) \left( \frac{e^{\gamma t}k^1(t) + k^2(t)}{2} \right)^\alpha - \delta. \tag{25}
\end{align*}
\]
A constant interest rate in country 2 implies that interest rate in country 1, and hence the growth rate of consumption, tends to infinity. This is incompatible with the existence of a long-run equilibrium. Conversely, interest rate of country 2 tends to zero if country 1’s is constant. This behavior permits the existence of a BGP in which physical capital of country 2 is equal to zero and its terms of trade strictly increase over time.

Taking into account these results, we find that country 1’s capital and consumption grow at the same rate $\bar{\sigma}$ as in autarky. Regarding proportions in (22), they become:

$$v^i(t) = 1 - \frac{\alpha}{2}; \quad v^2(t) = \frac{\alpha}{2}.$$  \hspace{1cm} (26)

International trade allows country 1 to increase its production of intermediate good 1 (its labor) in the proportion $\alpha$. Thus, country 1 owns total capital income and half of world labor income, while the rest of labor income corresponds to country 2.

Again, the conclusion is that trade acts as an engine of growth for country 2. Capital-augmenting technological progress in country 1 makes good 2 relatively more abundant than good 1, which results in a continuous improvement of country 2’s terms of trade:

$$p_1(t) = \alpha \left( \frac{1 - \alpha}{\alpha} \frac{p_1(t)}{p_2(t)} \right)^{t-u}, \quad c^2(t) = p_1(t) \rightarrow \frac{\dot{c}^2(t)}{c^2(t)} = \frac{\dot{p}_1(t)}{p_1(t)} \rightarrow \frac{\dot{c}^2(t)}{c^2(t)} = \bar{\sigma}. \hspace{1cm} (27)$$

The exported amount of good 1 by country 2 holds constant over time, but its imports are permanently increasing because the terms of trade become increasingly favorable to this economy. Hence, the role of trade volume is substituted by the role of the terms of trade movements. Country 2 does not need to accumulate physical capital for continuously raising its consumption level. The existence of non-reproducible factors is crucial for achieving this result, since it permits a strictly decreasing time path for the autarky relative price of country 1.
5. GENERALIZATION OF THE MODELS: TWO EXAMPLES

The models analyzed so far must be understood as simple theoretical structures that permit to affirmatively answer the first question. They also allow to separately identifying two channels through which trade can transmit permanent growth to an otherwise stagnant economy.

Our goal in this section is to show that the models can be extended to include more general technologies, other type of sectors and endogenous growth, while preserving the answers to both questions. In the next subsections, we propose two two-country models that generalize model 1 and 2. We will keep working under the assumptions of perfect competition and absence of international factor flows. We will also keep the assumptions regarding the structure of the population and the time endowment of household.

5.1. An Endogenous Growth Model with Leaning-by-Doing a la Arrow

Here, we sketch a model of endogenous growth with human capital accumulation that generalizes our model 1. The structure of the model is as follows. The world economy consists of two large countries, \( i = 1, 2 \), that produce two potentially traded goods: a consumption good, \( y_c^i(t) \), \( i = 1, 2 \), and a capital good, \( y_k^i(t) \), \( i = 1, 2 \), produced in country \( i \) and used in country \( j \). As usual, the omission of sub-index \( j \) denotes the whole production. Both goods are produced with capital, \( k^i(t) \), \( i = 1, 2 \), and labor with a Cobb-Douglas type of technology that exhibits constant returns to scale. Countries differ in the productivity of labor among sectors. In country 1, the efficiency units of labor, or human capital, accumulate through a learning-by-doing (LBD) process a la Arrow (1962). Human capital can be used in both sectors, that is, is general human
capital. In country 2, however, this learning process only takes place in the consumption good sector and, hence, it is sector-specific human capital. The next expression summarizes the main assumptions regarding technologies in an autarky situation.

**Country 1: Growing Economy**

\[
\begin{align*}
y'_1(t) &= (k'_1(t))^q (h'_1(t) l'_1(t))^{1-q} \\
y'_2(t) &= (k'_2(t))^q (h'_2(t)(1 - l'_2(t)))^{1-q} \\
k'_1(t) &= k'_1(t) + k'_2(t) \\
\dot{k}'_1(t) &= y'_1(t) - \delta k'_1(t) \\
\dot{h}'_1(t) &= \gamma l'_1(t) - \delta h'_1(t), \quad \gamma, \delta > 0 \\
y''_1(t) &= I'_1(t) = I'_1(t) + I'_2(t)
\end{align*}
\]

**Country 2: Stagnant Economy**

\[
\begin{align*}
y''_2(t) &= (k''_2(t))^q (h''_2(t)(1 - l''_2(t)))^{1-q} \\
k''_2(t) &= k''_2(t) + k''_2(t) \\
\dot{k''}_2(t) &= y''_2(t) - \delta k''_2(t) \\
\dot{h''}_2(t) &= \gamma l''_2(t) - \delta h''_2(t), \quad \gamma, \delta > 0 \\
y''_2(t) &= I''_2(t) = I''_2(t) + I''_2(t)
\end{align*}
\]

In country 1, the labor input in both sectors is equal to working time, \( l'_1(t) \) and \( l''_2(t) \), multiplied by human capital, \( h'_1(t) \). This country is represented by a version of the endogenous growth model developed by Arrow (1962) and, hence, enjoys sustained growth in autarky. In country 2 there is no labor productivity growth in the capital good sector. Human capital is only accumulated as a consequence of investment in the consumption good sector, \( I'_2(t) \). Since none of the reproducible factors accumulate with a technology that exhibits constant returns to scale in reproducible factors, country 2 cannot sustainably grow in an autarky situation. In other words, the condition for the existence of endogenous growth established by Rebelo (1991) does not hold.

The model can be solved to show that country 2 may overcome decreasing returns by participating in international markets. If country 1 and 2 had comparative advantage in capital and consumption goods, respectively, the latter economy stops producing capital good and permanently accumulates human capital. The trade equilibrium in the long-run will always be characterized by complete specialization, since the productivity parameter in human capital technology is the same in both economies. It is clear that the
greater the imported amount of capital good is, the more human capital is accumulated by country 2. The terms of trade of countries would become constant at long-run, since all technologies exhibit constant returns to scale in reproducible factors. The trade volume, however, would continuously rise and lead to the transmission of sustained growth to country 2.

5.2. An Exogenous Growth Model with Non-Reproducible Factors

To generalize our model 2 we propose a model of exogenous growth and trade in intermediate inputs. The crucial assumption in this case lies on the existence of non-reproducible factors. Countries produce a non-traded final good, \( y' (t), i = 1, 2 \), using two potentially traded intermediate inputs, \( x_z (t), z = 1, 2 \). The final good can be used for consumption or investment in capital. Intermediate goods 1 and 2 are produced with capital, \( k_i (t), i, z = 1, 2 \), and labor, \( l' (t) \) and \( 1 - l'' (t) \), respectively. Countries differ in labor productivity. More specifically, in country 1 there is labor-augmenting technological progress, which evolves to the exogenous positive rate \( \gamma \). Country 2 does not possess any kind of engine of growth. The next expression contains a summary of technologies in both economies.

\[
\begin{align*}
\text{Country 1: Growing Economy} & \\
y_i (t) &= \left( x_i (t) \right)^{a} \left( x_i (t) \right)^{1-a} \\
x_i (t) &= \left( k_i (t) \right)^{\theta} \left( h' (t) l' (t) \right)^{1-q} \\
x_i (t) &= \left( k_i (t) \right)^{\theta} \left( 1 - l' (t) \right)^{1-q} \\
k' (t) &= k'_i (t) + k'_z (t) \\
h' (t) &= \gamma h' (t), \quad \gamma > 0 \\
\text{Country 2: Stagnant Economy} & \\
y_i (t) &= \left( x_i (t) \right)^{a} \left( x_i (t) \right)^{1-a} \\
x_i (t) &= \left( k_i (t) \right)^{\theta} \left( l' (t) \right)^{1-q} \\
x_i (t) &= \left( k_i (t) \right)^{\theta} \left( 1 - l' (t) \right)^{1-q} \\
k' (t) &= k'_i (t) + k'_z (t) \\
\end{align*}
\]

The labor input is a non-reproducible factor, except in sector 1 in country 1 due to the presence of labor-augmenting technical change. Therefore, country 1 enjoys
permanent growth in autarky, while country 2 remains stagnant at long-run. It is worthwhile to notice that, in autarky, good 2 in country 1 becomes increasingly relatively scarcer, and thus more expensive, than good 1. In country 2, relative prices of intermediate inputs equal one.

Country 1 and 2 eventually have comparative advantage in good 1 and 2, respectively. The model can be solved using standard techniques to show that the free trade situation may be characterized by either complete or incomplete specialization of country 1. In both scenarios, the terms of trade of country 2 continuously improve, which results in the transmission of sustained growth. The stagnant economy overcomes decreasing returns because the continuous improvements in its terms of trade prevent the value of marginal productivity of physical capital from declining over time.

6. CONCLUSION

An important part of the new growth literature has focused attention on the link between international trade and growth. It has been argued that trade affects growth by impacting the extent of knowledge from abroad. To this respect, the crucial role played by technological spillovers has been extensively studied from both theoretical and empirical points of view. In this paper, we have gone further in analyzing the effects of trade on growth. In concrete, we have posed two related questions: might trade act as the sole engine of growth for an economy? If the answer is affirmative, how does trade operate in transmitting permanent growth? To our knowledge, the rather scarce theoretical literature on this issue has analyzed the first question, but no attempt has been made to answer the second one.

Here, we have answered both questions with two simple models of exogenous growth and trade of intermediate inputs. The models only differ in the type of technical
progress. More specifically, technical change is labor-augmenting in first framework, and capital-augmenting in the second one. The world economy consists of two large countries whose economies behave as Ramsey’s, but just one of them enjoys sustained growth in autarky. In both models, the stagnant economy imports growth by trading. However, the kind of technical progress matters for how growth is transmitted. In the first model, the mechanism of transmission is associated to permanent increases in the trade volume, while it relies on continuous improvements of the terms of trade in the second one.

These simple models have been useful for establishing the conditions under which trade emerges as the sole engine of growth for an otherwise stagnant economy. They also have permitted separately identifying the two channels through which trade can operate in transmitting sustained growth. We have shown that our simple theoretical frameworks can be generalized to include other assumptions, as more general technologies, endogenous growth and other kinds of goods, while maintaining the answers to the two questions.

REFERENCES


